ARTICLE VI.

SYMMETRY AND RHYTHM.

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It is a fault, or else an excellence, of human language that no word long remains perfectly unambiguous. We at first coin a word to express an idea, and presently either expand its meaning to cover kindred ideas, or contract it, and restrict it to a part of its original signification. Even in the mathematics, where, if anywhere, we should find words absolutely unambiguous, every symbol, every term, conveys more than one meaning, according to the connection of thought. The words "symmetry" and "rhythm" are, of course, no exception to this general law. Symmetry primarily, according to its etymological derivation, refers simply to equality of measure. But no man can consider seriously his own conception of symmetry, without discovering that he usually perceives in an object which he calls symmetrical something deeper and of more importance than a mere equality of dimensions. Hence we come to recognize two principal significations in the word. The first is a regularity of form which can be determined by compass and rule, consisting merely in the equidistance of points from some point, line, or plane of reference. The second meaning demands, in addition to this equidistance, — nay, even sometimes finds, in spite of failure to conform to the standard of rule and compass, — a higher quality, akin to beauty. This second meaning of symmetry contains an implicit recognition of geometric law. In like manner, rhythm refers primarily to a mere equal division of similarly recurring divisions of time; but in a higher sense asks that this regularly recurring division of time should be such as will produce agreeable sensations in him who perceives it. Before we can deal intelligently with our subject, we must therefore define clearly our ideas in relation to the laws of space and time. Symmetry does not refer, in its best sense, simply to material things, but to the space which they occupy. The Grecian architects discovered that a perfect conformity of material things to symmetrical forms did not suggest those symmetrical forms to the beholder so distinctly as they were suggested when the marble was made to deviate slightly, according to perspective laws, from the form which it was intended should be embodied in imagination by the beholder. The attempts of civilized governments to secure uniform weights and measures, in conformity to a single standard, show us how difficult, how impossible, to make a material substance exactly in conformity with an ideal measure. The objects which we call symmetrical we call so only because they suggest to our imagination symmetrical forms. Yet those forms exist not only in our imaginations, but in space also, and, out of our imagination and thought, in space only. And by space I mean that which the ordinary, common sense view of mankind understands by the term; not that which modern philosophers, or would-be philosophers, mean, but that which Plato meant — an incorruptible, eternal entity, having no other properties than its three dimensions, of boundless extent, and incapable of division or separation except through an act of intuition or imagination. The dimensions of a material thing arise from the property in matter of occupying space. The idea of space in the mind is not innate, but arises from the instantaneous perception of space by the intellect, on the occasion of the first perception by the senses of a body occupying space.

Space, being infinite in extent, and indivisible except in imagination, is handled by the human reason only through the means of negations; that is to say, the boundaries of its divisions themselves occupy no space; they are positive to thought, but negative, or rather zero, in regard to space. The first of these zeros is a surface; the second is a line; the third and most complete zero is a point. A point is an absolute negation of dimensions, but is located, or has position, in space. It is one of those contradictions in which the human mind finds itself involved, whenever it attempts to fathom the depths of its own thought, that we should thus speak of the position in space of that which does not occupy space. An equal inability is felt by the mind in attempting to locate a single point in space without referring to another point, or, indeed, to locate any body in space without referring to other bodies. We pass from points to lines in one of two methods, both inconceivable, and yet both necessary, in geometrical reasoning. We may define a line to be a series of points infinitely near together, but in which each is contiguous only to two others, and they on its opposite sides. The inconceivability of this conception lies in the point's having no dimensions; so that however nearly contiguous two points may be there is room for an infinity of points between them, and however infinite the number of points may be, since they have no dimensions, they cannot make length to a line.

The other mode of obtaining a line from a point is Newton's. It defines a line to be the path of a point moving in space; and its inconceivability lies in the fact that a point has been defined to be a mere position in space, which cannot be moved, although the attention may be transferred to other points.

We obtain the idea of a surface, in like manner, from a point, in two distinct ways. A surface may be defined as a series of points contiguous to each other, arranged in such a manner that they shall occupy no space, and yet, at each point, a series of points shall extend in every direction which is at right angles to a certain line passing through the chosen point. Or a surface may be defined as the path of a moving line, when the points in that line are not, by the motion of the line, moving in the direction of the contiguous points in the line. These modes of generating a surface have in them the same elements of inconceivability as the modes of generating a line. A solid is simply a portion of space inclosed by a surface; and inasmuch as space is homogeneous, the geometrical properties of a solid are discussed in the discussion of the surface which incloses it.

The human mind has no interest in the discussion of points, lines, and surfaces, unless they conform to law,— unless they are obedient, as it were, to the mind. This figurative expression, obedience to law, may be best explained, perhaps, by Newton's conception of a line being generated by the motion of a point; although it may be justified, also, in the other way of looking at the composition of lines and surfaces. When a line, for example, consists of a series of mutually contiguous points so arranged that the position of each and every one may be defined by defining the position of any one, then we evidently may say that all these points are subject to one and the same condition of position. It is not a harsh figure to speak of them as obedient to that act of thought which defines the conditions of their position. If, now, we conceive, in Newton's fashion, a point as running through such a line, we may say that the point everywhere moves in obedience to one law — the law which defines what its position shall be at each part of its path. When we remember that a line extends only in one direction, and is absolutely without dimensions in the other directions, we see that this law which governs the motion of a point is exceedingly strict; it permits motion in the line with perfect freedom, but absolutely forbids any motion whatever out of the line. Both of these methods of explaining or justifying the figurative expression "obedience to law" may be applied to surfaces, as well as lines. A surface is a series of contiguous points, so arranged that we may, through any one of them, draw a straight line which shall be at right angles to all lines joining that point to contiguous points of the surface. That surface may be said to conform to law when each point in it is subject to one and the same condition of position; so that a point or a line moving in that surface may be considered as obeying in its motion always one and the same law, namely, the law or expression of thought which defines its conditions of position. And the law of surfaces is equally strict with the law of lines; the moving point is perfectly free to move in the surface, and absolutely forbidden to leave it.

It will readily be seen that these conditions of position must of necessity be intellectual conditions, since the parts of space have no property by which they can be distinguished from each other in any other way than by a mental act. We may express these conditions by language of a greater or less degree of accuracy and strictness, or we may represent them by diagrams, drawings, or models. Thus the circumference of a circle is a familiar term, sufficiently understood by every educated person; and its law may be expressed in various forms of ordinary language, or by geometric definition, or by equations of analytical geometry, or by a drawing or figure sufficiently accurate to suggest the exact idea. It will readily be seen that no circle made by human art of a material substance can be absolutely accurate, although in the best mathematical instruments an exceedingly close approximation is obtained. But the conception of a perfect circle is very readily awakened in the mind of the beholder by a diagram or drawing very far from exact accuracy. Many other figures which have been investigated, and expressed in the technical language of geometry and algebra, are also familiar to the minds of all educated persons; and a very precise idea of them is readily conveyed either by the forms of ordinary language, or by drawings which give only an approximate expression of their laws. There are also many beautiful figures, familiar to our eyes, which have not been analyzed, described, or defined in technical language, but which we instinctively feel must be capable of such expression under some future, or at least conceivable, increase of mathematical skill. Thus the varying forms of crystals distinctly suggested to every student the faith that they were subject to law, for a long period before the law was actually discovered and expressed in the modern systems of crystallography.

Thus, also, the botanists had perceived that the leaves of plants are arranged in a kind of symmetry around the stem long before Peirce showed that they obeyed the simple law of division in extreme and mean ratio.

Symmetry in its higher meaning implies this obedience to law. The symmetical form is that which not only has equal dimensions in sundry directions, but that which is inclosed in lines, or surfaces obedient to law. This distinction between the better idea of symmetry and the inferior, has not been clearly made by many writers; but its reality has been felt, I suspect, by every student of nature or of art. A waving line made by equal arcs of circles, turning alternately to one side or the other, and having common tangents at their points of junction, is symmetrical in the lower sense; but it suggests, to every eye keenly appreciating beauty, a more beautiful line, the elastic curve, which obeys everywhere a single law, and does not, like the alternating arcs of circles, change its law when it changes its direction of curvature.

The sense of beauty is awakened by the embodiment of a simple law giving rise to a varied outline; and the degree of beauty felt by a cultivated beholder is in direct proportion to the simplicity of the law, and also to the variety, or complexity, of its manifestations. Another element enters into the formation of beauty, an element which makes a transition to the idea of rhythm. When in the consideration of symmetry we take Newton's notion of fluents and fluxions, and consider surfaces and lines generated by motion, we leave the consideration of pure space, and introduce the idea of time also. The motion of a point, at a uniform rate of speed, through a line obedient to law, must, must of necessity, be governed by rhythm, as well as by symmetry. In this conception of rhythm, the idea of number is much more prominent than in the conception of symmetry alone. By the definition of beauty which we have just given, both rhythm and symmetry should be more beautiful in proportion to the simplicity of the numerical ratios entering into them, and to the multiplicity of their combinations. Thus the law of phyllotaxis to which I have just alluded, embodies only the three smallest prime numbers, two, three, and five, but embodies them in such a way as to admit of unnumbered forms of manifestation.

The beauty of proportion has been conceded by architects and by painters, although it has not been clearly defined by them. In Hay's analysis of Grecian architecture he shows that beauty of proportion probably consists in the numerical ratio of angles to a right angle. The angles whose proportion constitutes the beauty need not be distinctly marked by lines, but must be strongly suggested to the imagination. Thus to take the simplest case, a rectangle is of fine proportion, when a diagonal would divide the right angle in the proportion of prime numbers smaller than ten. I have myself subjected this supposed law of Hay to independent experiment. Making a rectangle variable in its proportions, I have asked persons of good taste, ignorant of my design, and never having heard of Hay's law, to arrest my alteration of proportions in the rectangle at the instant when they deemed them most beautiful, and I have found that, in an overwhelming majority of cases, they stopped the movement of my hand at precisely the point where an imagined diagonal divided the right angle in the proportion of one to two, else in that of two to three. Such experiments tried upon a great variety of persons, both adult friends and pupils in high schools and colleges, have demonstrated to me the fact that the eye of persons having a taste for drawing and architecture, recognizes unconsciously, if I may use so paradoxical an expression, the existence of numerical harmony in the magnitude of suggested angles. A similar and still more extensive series of experiments which I have made in public schools, and in private circles, has convinced me that the child of civilized parents, to say the least, recognizes in music not only that numerical harmony which is found in the rhythmical movement of a composition, but also that hidden numerical harmony and proportion of ratios which lies concealed in the nature of musical tones, melodies, and progressions. Arrangements having been made by which the members of a large school should write down independently of each other, their impressions of the aesthetic meaning of a piece of music, heard by them for the first time, I have found that three quarters of the pupils were in substantial agreement concerning the precise shade of feeling which the music conveyed, and which I knew it was intended by the composer to convey.

Scientific men no less than artists have an implicit recognition of the existence of law and harmony in the external world. The history of progress in every science bears witness to this fact. Observers of nature have never been content with the mere observance of the facts of nature, but have always sought to group those facts together, and link them to each other, by intellectual bonds. Moreover from the earliest period there has been manifested, in the history of scientific progress, an invincible faith among scientific men that the facts of nature are capable of being arranged in conformity with intellectual laws; laws of geometry and algebra. Nor has this faith proved deceptive. In many cases centuries of observation and of patient study have been requisite in order to discover those relations in external things themselves, which correspond with or conform to symmetrical laws in space, or rhythmical movements in time; but whenever science has risen above empirical formulae to rational, scientific formulae, the victory has been accomplished. The origin of this conformity of external nature to the intellectual laws of space and time has not always been inquired into by the scientific mind; that is not a scientific problem. The scientific instinct prompts man to seek the law, and may lead him to a successful search, without even suggesting to him the question of the origin of the law. This latter question is suggested by a different, a metaphysical or religious instinct; and may be wisely and successfully pursued by one whose acquaintance with the results of scientific inquiry is but slight. No possible extension of the results of the investigation of external phenomena can affect the question of the origin of those phenomena; it may affect the question in what manner, in what order, the phenomena have arisen, but it cannot go back to the origin itself.

As a matter of fact no man confines himself exclusively to his speciality; whatever a man may devote himself to, as a profession, he has also some general culture, and is interested in the general questions which interest mankind. Indeed the interdependence of all branches of human knowledge makes it usually true that a man cannot become a successful student of a special department unless he lays a broad foundation, in a general knowledge of matters of human interest. The structure of scientific attainment is most secure, and can be raised to its loftiest height, only when its foundations are as wide and as deep as the individual character of the student's mind will permit. From this fact, that no man can be, even if he try, a pure specialist; and from the fact that a general culture is requisite in order to give the highest success in special pursuits; it follows that scientific men must speculate to a greater or less extent upon theological subjects; that men devoted to the natural sciences must have some knowledge of pure mathematics; and that theologians and mathematicians must peer into the laboratories of the physicist and the biologist, with liveliest interest. Men are not to be reproved for thus going "beyond their last"; they are not to be discouraged from thinking, and even speaking, upon subjects which lie out of their special sphere; only provided, that they retain a proper humility, and do not attempt to decide dogmatically upon questions, to which other men have given much more attention; with perhaps an equal or superior native ability to understand. But let it be recollected that the man of physical science is just as liable to misconceive and misunderstand a theological question, as a theologian to misconceive facts of natural science. The psychologist, mathematician, and theologian require no laboratory, they are unable in general to put their hypotheses to physical tests; but it does not follow that the investigations of the geometer and the metaphysician, and of the divine, require no intellect and no special training. W. R. Hamilton's Quaternions, and W. Hamilton's posthumous lectures, make as real a demand upon the reader, for intellectual power and intellectual training, as can be made by any writing of Helmholtz or Darwin. The effect of true culture ever must be to remove self-conceit, as well as to give confidence in the results of learning. The effect of a liberal education ought certainly to be a cultivation of the spirit of respect for all intellectual workers in whatever field.

In regard to the question of the origin of natural law, there have been mistakes by both great parties for whom these questions have most interest. The metaphysician and the theologian have frequently shrunk from the acknowledgment of the supremacy and universality of law, as though such a view would destroy human freedom, and make the world, as they express it, a mechanical, brazen universe. In the Duke of Argyll's "Reign of Law" he shows the fallacy of this judgment, and illustrates, in a varied and beautiful manner, how the presence of intellect may be made even more manifest in a universe governed by law. On the other hand, many scientific students have misconceived the theological position that all the universe is dependent upon the will of God, thinking that position inconsistent with the universality and invariability of law in the universe. The theologian admits no such incompatibility; since he regards the will of God as a power guided by absolutely infinite wisdom — a wisdom which foresees from eternity the best possible mode of action in all the absolutely infinite variety of cases which can arise. Indeed, the theologian claims that it is from his science that the idea of law in the universe can alone be legitimately deduced. Historically, it is, I believe, true that the earliest statements of the universality of law are found in connection with religious thought. To one whose mind has grasped the conception of the existence of an infinite Deity, of infinite wisdom and power, the whole universe is but the expression of a single thought, producing an infinite variety of detail, but also possessing a perfect unity. In this theological form the doctrine of the correlation of forces and of the consequent inter-dependence of all sciences, was familiar to metaphysical and theological writers long before its tardy confirmation, or partial confirmation, by the induction of physicists. And it appears to me that the popular modern school of Darwinians are in reality wandering far from this faith in the universality and invariability of law, to which they will be brought back, if brought at all, only by a closer metaphysical analysis of the postulates of science.

The natural sciences have, like the mathematics, their postulates; and among them is that of the invariability of law. It seems to me that they also require the postulate of the universality of law. In the introduction to his "Leben Jesu," Strauss endeavors to demonstrate both postulates by an appeal to a still higher axiom — the infinity of God. God being infinite, he says, can act only simultaneously throughout all space, and synchronously throughout the past and future eternities; that is to say, he can act only by universal and invariable law. The objection to this demonstration of Strauss lies in the form of necessity which he gives to his conclusion. By thus denying the ability of the Divine Being to act in any other than certain ways, he limits and destroys the very infinity from which he professes to draw his conclusion. But when we assign to the infinite Being the attributes of infinite knowledge and wisdom which even by Spinoza's pantheism we are compelled to do, we are ready to admit the conclusion of Strauss, with a limitation; the Divine Being will act by universal and invariable laws in every instance in which it is possible for such laws to accomplish the best ends. We cannot retain our intellectual clearness and unity of thought, if we admit that even infinite power can make finite contradictories both true. This is not putting a limitation upon infinite power, but simply asserting, although in a negative form, that God's wisdom also is infinite.

Before proceeding to show in what manner the philosophy of Erasmus and Charles Darwin denies the universality and invariability of law, it may be better to expand a little more fully our definitions of law. I have already said that a geometric law expresses the position of each one of an infinite number of points by defining the position of one point; in other words, that a series of points is said to be obedient to law when each one of the points is subject to one and the same mental condition of position. These mental conditions of position are always capable of algebraic or geometrical statement. In these statements of the condition of position two kinds of quantity are introduced — constants and variables. The variables express the relation of the position of a point to some fixed point, line, or plane; the constants express the manner in which the variables are connected in the law. It will thus be perceived that changes in the variables produce no change in the law, but simply take us from one point to another of a line, or surface, which obeys the law; while, on the other hand, changes in the constants make a change in the law itself. The magnitude of a change made by this change of the constants depends very much upon the nature of the law; for example, to change the constants in an ellipse changes the figure from a circle to a longer oval, or even to a parabola or hyperbola; while if we take a new form of the law of a parabola, we can no longer make it revert into an ellipse, but may, by the change of a single constant in it, alter it into a catenary, or into an elastic curve. The mathematicians have devised means of tracing all the possible variations of a law which cannot ordinarily be made by changes in the constants. If the number of constants is small, the number of changes is confined to a small number of cycles which may be thoroughly investigated. If, on the other hand, the number of constants is more than three, the number of variations in the law becomes much greater, and may be practically unlimited. For example, in the Ptolemaic system of the universe each planet was supposed to be carried upon the end of an arm rotating about the end of a second arm, which in its turn rotated about the end of a third, and so on. By varying the length of these arms and the velocity of their rotations, the planet can be carried in any orbit in which observation shows that it is moving. This Hipparchian device of successive deferents had been anticipated, by many thousand years, in the formation of the human arm, in which the humerus is a deferent rotating about the shoulder, the ulna rotates at the end of the humerus, and the metacarpal and phalangeal bones give, finally, to the fingers' end, the ability to move in any curve whatever. Thus to the human fingers is given absolute liberty of motion, yet in obedience to this epicyclic law, which ties them to the socket of the shoulder. But the instinctive faith of the scientific man always leads him to expect in the laws of nature, by which the movements of the physical universe are governed, a much greater simplicity. The expectation has sometimes been, as I have already said, long cherished before it was fulfilled. The careful observations of Chaldean astronomy were not satisfied by the discovery of the law, until Kepler, more than thirty centuries afterward, justified the labors and faith of those ancient astronomers. In this case, not only were the observations made many centuries beforehand, but the law itself, of the ellipse, had been prepared by the labors of the Greek geometers more than fifteen hundred years before it was discovered that it governed the motions of the heavenly bodies; and thus a law involving two constants was substituted for the epicycles involving twelve or fifteen. The history of science does not contain many so striking illustrations of the long interval which may elapse between the physical observations and the interpretation of them by the mathematician; but it contains innumerable examples of cases in which observers had almost despaired of the discovery of the law, or had thought that the law if discovered must be exceedingly complex, and then were surprised and rewarded for their patience by suddenly finding an exceedingly simple law governing all the intricate phenomena. We have, therefore, not only in the simple, ineradicable scientific instinct, which looks for law and order in the succession of physical phenomena, but also in an induction from the history of numerous scientific discoveries, a basis for our belief that in every series of phenomena a patient investigation will finally reveal a simple law. If in our day we repeat the Hipparchian and Ptolemaic experiment, making a complicated hypothesis, involving numerous constants, to explain a group of phenomena, — if we do this for any other purpose than a mere convenient mnemonic by which to bind the facts together, we are not only disregarding the scientific instinct, but also the warnings of the history of science.

But the scheme of evolution proposed by Erasmus Darwin, and made popular by the immense learning and agreeable style of Charles Darwin, seems to me to be precisely of this character; it is a virtual denial of the existence of law in a department in which the whole intellectual history of science would lead us most surely to expect the presence of law. The doctrine of the evolution of one organic form out of another has in itself an antecedent probability, It is, as Erasmus Darwin has said, more consonant with our ideas of the infinite wisdom and power of the Creator to suppose that he has made a universe capable of self-evolution. It evidently requires a higher power to create a plant or an animal capable of continuing its kind by reproduction, than to build the most ingenious machine incapable of multiplying itself. In like manner, it would seem to us a higher product of divine skill to find an animal capable not only of reproducing its kind, but of giving birth to new kinds, capable, again, of increasing and improving the nature of their descendants. Dr. Darwin expresses this probability by saying that it requires a higher infinity to create a cause of causes than to create a cause of effects. But we are not to build our theory of the universe upon *a priori* conceptions alone; we are carefully to observe the facts of nature. When we do reason from first principles, and from our conceptions of the attributes of God, we are carefully to test all our processes not only by the laws of logic, but by a comparison of their results with the facts of the external world. I have heard one of the finest living mathematicians argue with faultless logic for half an hour to arrive at the conclusion that the solar tides are greater than the lunar. Nothing but the inconsistence of this result with the notorious facts of observation led him to review his mathematical calculations, and to discover its error, singularly enough, in the very first axiom which he assumed in the beginning. The doctrine of evolution assumes the possibility of evolution; and certainly this assumption cannot be considered as axiomatic. Nevertheless, it is not the doctrine of evolution which I regard as being in conflict with the universality of law; it is only the form of that doctrine adopted by the Darwins. The offensiveness of the form does not appear so distinctly in the writings of the grandfather; it is the grandson who sets forth its marked opposition both to teleology and morphology. The doctrine of teleology I have defended in this journal, and do not propose to touch upon it at the present time; it is from the morphological side alone that I wish to show the improbability, if evolution has taken place in the organic life of the globe, of its having taken place through the variations of species and the "survival of the fittest."

The form of the ultimate atoms of matter, if it consist of atoms, as is now usually assumed, may be wholly unknown. But the molecules of almost all substances known to us are evidently of regular form. The chemist and physicist suppose that even those elements which seem in our present knowledge amorphous in their molecular constitution, may, under some future torture of experiment, reveal themselves in crystalline form. The minute molecular motions which produce what we call the secondary qualities of matter are all rhythmical in their character; the molecules, therefore, obey law in time and in space; hence they probably obey law in their dimensions. The atomic weights of elements are not manifestly governed by law; and yet Professor Pliny E. Chase has shown from spectroscopic analysis, the probability that they are arranged in laws of harmonic proportion.

Certainly, the combinations of molecule with molecule are governed by simple numerical ratios. The cosmic masses of matter, it is well known, are obedient, in their larger motions, to law; and even the form of the apparently irregular upheavals of the crust of our earth is shown, by Peirce's acute observation on the direction of long coast lines and long mountain ranges, to have been controlled by the crystalline formation of rocks. When we come to examine organic matter the same symmetry and rhythm become even more apparent. The organic world is a system of time-keepers; each creature has periodic functions; it responds to the revolution and rotation of the earth and of the planets. And this periodicity is even more marked in the higher than in the lower organizations. The movements of organic bodies are rhythmical; their forms also are symmetrical. The form of a plant or of an animal is governed by geometric law, as really as is the form of a crystal; the force which regulates the growth even exhibits strong symptoms of polarity such as is manifested in the molecular movements of unorganized matter. But there is a much higher symmetry in the plant and animal than in the crystal; new constants seem to be introduced into the law of its formation, which also by their variation give a still greater freedom of arrangement to the expression of law. In the highest plants and animals this symmetry of form and rhythm of movement give us our most perfect realization of the idea of beauty. The human imagination has never been able to exceed in its ideals of beauty the forms which are suggested and almost perfectly realized in the bodies of men and of the highest plants and animals.

The laws of symmetry and rhythm which govern the inorganic world were not revealed to the human intellect until after many centuries of patient observation and laborious comparison. Nor have they yet been fully laid open to the gaze of science. Physical investigations are pushed with more fervor to-day than ever before, and are yielding daily richer returns to the explorer. The laws which govern the form of organic beings will probably require even more assiduous observation and more laborious comparison; but they must finally be revealed to the patient student. But the complete expression of law requires algebraical and geometrical language; and the students of organic being can never attain complete success, until they have called in the assistance of the mathematician. Thus the botanists had given a very imperfect representation of the law of phyllotaxis until, less than thirty years ago, Peirce showed that it was the law of extreme and mean ratio; then Chauncey Wright proved that this ratio gave the most equal distribution of leaves about the stem; and Dr. Hilgard ingeniously connected it with a law of development of cells. Zoölogy had made many attempts at the classification of animals, and Agassiz had shown that the most varying schools in that science attained substantially similar results; but Peirce, in his lectures on analytical morphology, has alone given a foundation of mathematical certainty to any part of their scheme, by showing that the physical condition of the earliest embryo requires it by mechanical principles to take substantially one of four forms, by a mechanical necessity as imperious as the true form of an arch to enable it to sustain its own weight.

Since law thus pervades the world, from the molecule up to the highest organized being, it is an induction of the highest probability that law governs the classification of plants and animals — that Peirce's reduction of Cuvier's four branches to mechanical conditions of equilibrium is but the precursor of geometrical and algebraical generalizations which shall hereafter give to classes, orders, families, and genera the like clear, sharp definition. This, evidently, has been the strong, though vaguely defined, faith of most of the zealous students of the classificatory sciences in all ages of the world. They have with one voice condemned those who laid out an *a priori* system, and endeavored to arrange the facts of nature under it. They have evidently believed that in their classification of plants and animals they were describing external facts. Whewell, in his Philosophy of the Inductive Sciences, supposes ideal types of species to be described, and the individuals to be assigned to the type which they most nearly resemble. Of course, this is practically the mode in which plants and animals are classified; but in the mind of the botanist and zoölogist there had generally been a hidden feeling that the creatures themselves are as distinct as the ideal types; indeed, the idea of the type is but derived from the creature.

But under Charles Darwin's teaching all this is rapidly being changed. The naturalist begins to doubt whether there are any lines of demarcation in nature other than have been produced by the dropping out, by the annihilation, of a part of a series. He says that if we give him specimens enough of two or three closely allied species he can so arrange them that it shall be impossible for us to decide where one species ends and another begins. The like ambiguity is affirmed with regard to genera and higher divisions. I grant the fact; but I deny the conclusion that there is no distinction between the groups in nature. Much more emphatically would I deny that the fact points to a gradual evolution of one form from the other. I grant the fact only for the sake of argument; for I believe that it is not strictly a fact. I I think that an eye quick to detect botanical and zoölogical characteristics needs only a little more patient training to enable it to distinguish between specimens most closely resembling each other. There are many differences distinguishable to our senses which we nevertheless cannot describe with any sharp definition by words. Thus the difference between the apple and the pear, between the plum and the cherry, between the acorn and the chestnut is apparent to our senses, but is almost, if not quite, inexpressible in any terms of botanical science.

The complementary truth also holds, that the intellect can sometimes very clearly distinguish things indistinguishable by sense. Thus the elastic curve has, at the extremities of its series of variations, a straight line and a circle; and these straight lines and circles are indistinguishable to the eye, and even to the imagination, from the straight line and circle which we can produce by varying the eccentricity of an ellipse. Yet reason, which in the hands of the geometer transcends even imagination, shows that the two circles are entirely different, and also the two straight lines — that they cannot be considered alike, except by a process which would confound all intellectual distinctions, and destroy the possibility of science. The ellipse and the elastic curve belong to genera very far removed from each other — much farther than the oak from the chestnut; yet each in its variations may reach the form of a circle.

The observer unacquainted with mathematics might think that the ellipse could pass through the form of a circle into an elastic curve, and *vice versa;* but he cannot induce the geometer to consider it possible. When the forms of the oak and of the chestnut are as thoroughly understood by the botanist as two thousand years of labor have rendered the forms of the ellipse and the elastic curve understood by the geometer, the botanist will probably be able to make a clear statement of the difference between them, and will wonder that they were ever thought indistinguishable.

I look upon both questions as problems in geometry. The classification of plants and animals proceeds fundamentally upon differences of form or shape, and takes questions of magnitude, color, longevity, temperament, chemical nature, physiological peculiarities, only into secondary consideration. But every question of classification of forms is a geometrical question, and is to be decided by geometrical canons. The botanist and zoölogist may rebel, but they will rebel in vain. The numbers of Pythagoras and the axioms of Euclid are inexorable. The fates themselves cannot violate the laws of arithmetic and geometry; much less can fluttering theorists, however large their balloon, break through those adamantine bars. The erratic genius of DeMaillet and of the Darwins has built a plausible and ingenious theory, which, if I understand them, proposes to give up the problem of classification as unreal and impossible. According to this theory, when carried out to its fullest extent, there is but one kind of matter, which remains tolerably stable in its properties, but is subject to a very gradual change. When some portion of it has changed into a condition in which it chances to be stable and useful, the change is partially arrested. Thus out of the one element, some sixty have been formed. As these encounter and mix they form passive compounds, indefinitely various. Those which chance to have a more stable constitution remain, and we get minerals. But, in the muddy turmoil of the elements, a very peculiar compound, curiously unstable, was once formed, which has proved marvellously stable in its instability. This was a living germ. It multiplied; it took various forms; it became a plant; it became an animal; it developed the senses, perfectly in the animal, imperfectly in the plant; it developed sex, perfectly in the animal, imperfectly in the plant; it finally developed consciousness, intellect, conscience in man. All this was done by insensible gradations, so that if we could only have the complete line of generations, from Caucasian man back to the minute microscopic germ which first appeared on the planet there would be no break anywhere. Englishmen a thousand years ago were very much what they now are; ten thousand years ago they were not sensibly inferior to their present condition; a hundred thousand years there might possibly have been a slight inferiority in their physical powers; a million years ago they were still men, much higher than any present existing ape. Thus the change is very slow, and the time when the present species of animals began must be millions of years ago. The time since life began on the planet must at this rate be uncounted myriads of millions of ages. But, said I to an earnest Darwinian, the physicists are not inclined to allow more than one hundred millions of years since the earth was red hot over its whole surface. "So much the worse for the physicists," replied he. Such is the confidence of this school that their non-solution is the only solution of the problem of classification.

I call it a non-solution, because it denies the existence of any order or plan in the variations of organic structure. Evolution says that the development of the plan of the organic kingdoms was accomplished under the pressure and stimulus of the inorganic surroundings of the creatures. But Darwinism, affirming the development to have been by insensible gradations, denies, in effect, the existence of a plan. The appearance of a plan arises in their view simply from the gaps making apparent divisions between species where there are none. Classification is in their eyes merely a matter of human convenience; it is applied to the organic kingdoms in the same way in which the modern physicist applies the epicycloidal curves of Hipparchus to the curves of temperature, or barometric pressure. Whatever be our judgment concerning the reality of evolution, it seems to me in every way improbable that evolution has taken place, or is taking place, in any lawless or haphazard fashion; making all the labors of classification as empty of intellectual meaning as though they had been expended upon the forms of flying clouds, or the disposition of the settlings in cups of tea. The tone of the whole Darwinian literature is that of a retreat; it virtually calls upon the botanist and zoölogist to give up any further attempt upon the problem of classification; declaring the fortress in which the secret is intrenched to be invulnerable.

Agassiz's Essay on Classification, and his Methods of Study are, on the contrary, trumpet-calls to a forward movement, and awaken enthusiastic hope and firm resolve in the heroes who lead them. They call on the naturalists to press forward in the same direction in which the army has been moving since the days of Aristotle. Agassiz shows that, from the earliest Greek to the latest French and German efforts, there has been a virtual agreement among all students concerning the main divisions of the animal kingdom; this agreement has, in regard to a large portion of the kingdom, extended even to the details of species and genera. This substantial agreement in the boundaries between the divisions, he thinks, argues the possibility of a future agreement as to the intellectual grounds for placing the boundaries as they are; and he proceeds to indicate the character of those grounds as they appear to him. Agassiz was peculiarly fitted for this work; not only by his marvellous knowledge of details, in which he was the peer of Darwin, but by his breadth of philosophical views, in which he was far superior, as it seems to me, to any of his noted contemporaries. In the university he had passed, with honor, a searching examination on the philosophy of Plato, learned from Plato's untranslated writings; and this had qualified him for these higher walks of his own specialty. Deficient in sharpness of self-conscious psychological observation, he was pre-eminent in quickness and accuracy of external sense; and pre-eminent in the rapidity, breadth, and soundness of his generalizations from external observation.

His principles, in the form laid down in the Essay on Classification, may not stand the test of a careful comparison with facts, but they will, at some day, be slightly modified and reannounced by some grateful student; and will then stand the test. For their final vindication they will require a mathematical enunciation. This has been the destiny of other physical sciences; and the analogy of the physical sciences leads us to suppose that it must be the goal of biology also. The vagueness of arbitrary variation and survival of the fittest is a poetical dream; it must give way to the intellectual, scientific sternness of invariable law bounded by invariable conditions. As the four primitive forms of the embryo flow from necessary mechanical conditions, inflexible as the law of equilibrium in arches, so the classes, the orders, families, and genera are doubtless formed by the operation of sharply defined conditions, — sharply defined in nature, although not yet defined in human thought.

Such seems to me to be the teaching of the history of the inductive sciences; and such the instinctive faith of the most truly scientific and courageous naturalists and mathematicians. The geometer will not willingly relinquish the hope of great triumphs in the future; when the new methods of the nineteenth century shall be as faithfully applied to the problems of organic form, as the methods of the seventeenth have been to those of inorganic matter. If the naturalists retreat at the sound of the Darwinian bugle, and the trumpet of Agassiz fails to rally them, the mathematicians will press forward and win the field.

It is scarcely necessary to say to readers of the Bibliotheca Sacra that, with faith in the wisdom and truth of the Creator, to re-enforce geometric and scientific instinct, it becomes still more evident that the rhythm and symmetry of the organic kingdoms is not the result of accidental arrests and erasures in the remains of an insensibly slow variation. I have heard a deluded girl drum upon the piano, infatuated with the belief that she was possessed by the spirit of the mighty Beethoven; but in that monotonous succession of the simplest chords there was no rhythm in the higher sense; nothing to indicate the presence of any but the feeblest intellect, and the most uncultivated taste. I have, on the other hand, heard an indifferently trained orchestra play, I knew not what, except as the sounds themselves told me, and I instantly recognized the work as the work of a great mind and a noble heart. The rhythm and harmony of the organic world reveal the power, the wisdom, the love, of an infinite God. Even the system of Darwin, if it can be called a system, admits, by its doctrine of the survival of the fittest, the fact that each creature is, in general, fittest for the place it occupies; and only endeavors, with strange inconsistence, to show here and there a maladaptation. It is in poor taste, and less wisdom; it is worse than it would be for some sciolist in music to attempt to show that Bach's music did not come from a master mind, because here and there a passing note is introduced in a melody without connection with the fundamental harmony. It is better for the young student in music to assume in all his studies that Bach was a profound master, and that his works are the best models for harmonic effects. And so long as man is less than the universe in which he dwells he may safely assume that all is well and wisely put; and that it is his wisest and best course of study to seek everywhere, in every department of nature, not for discords and maladaptions, but for harmonies, correlations, adaptations. The universe is the sum of all symmetries, and contains all geometries, architectures, sculptures, and pictorial arts. It is the sum of all rhythms, melodic or harmonic, and contains all algebra, poetry, music, and dance. The divine Word which created it is wisdom and love, and manifests wisdom and love in every syllable and tone in which it utters itself; not least in the wondrous series of the forms of plants and animals, swaying in the responsive rhythm of growth and decay, sleep and activity, generation and succession, to the periodic march of the planets, the moon, and the sun.